

14-1 Study Guide and Intervention

Trigonometric Identities

Find Trigonometric Values A **trigonometric identity** is an equation involving trigonometric functions that is true for all values for which every expression in the equation is defined.

Basic Trigonometric Identities	Quotient Identities	$\tan \theta = \frac{\sin \theta}{\cos \theta}$	$\cot \theta = \frac{\cos \theta}{\sin \theta}$
	Reciprocal Identities	$\csc \theta = \frac{1}{\sin \theta}$	$\sec \theta = \frac{1}{\cos \theta}$ $\cot \theta = \frac{1}{\tan \theta}$
	Pythagorean Identities	$\cos^2 \theta + \sin^2 \theta = 1$ $\tan^2 \theta + 1 = \sec^2 \theta$ $\cot^2 \theta + 1 = \csc^2 \theta$	

Example Find the exact value of $\cot \theta$ if $\csc \theta = -\frac{11}{5}$ and $180^\circ < \theta < 270^\circ$.

$$\begin{aligned} \cot^2 \theta + 1 &= \csc^2 \theta && \text{Trigonometric identity} \\ \cot^2 \theta + 1 &= \left(-\frac{11}{5}\right)^2 && \text{Substitute } -\frac{11}{5} \text{ for } \csc \theta. \\ \cot^2 \theta + 1 &= \frac{121}{25} && \text{Square } -\frac{11}{5}. \\ \cot^2 \theta &= \frac{96}{25} && \text{Subtract 1 from each side.} \\ \cot \theta &= \pm \frac{4\sqrt{6}}{5} && \text{Take the square root of each side.} \end{aligned}$$

Since θ is in the third quadrant, $\cot \theta$ is positive. Thus, $\cot \theta = \frac{4\sqrt{6}}{5}$.

Exercises

Find the exact value of each expression if $0^\circ < \theta < 90^\circ$.

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|---|---|
| 1. If $\cot \theta = 4$, find $\tan \theta$.
$\frac{1}{4}$ | 2. If $\cos \theta = \frac{\sqrt{3}}{2}$, find $\csc \theta$.
2 |
| 3. If $\sin \theta = \frac{3}{5}$, find $\cos \theta$.
$\frac{4}{5}$ | 4. If $\sin \theta = \frac{1}{3}$, find $\sec \theta$.
$\frac{3\sqrt{2}}{4}$ |
| 5. If $\tan \theta = \frac{4}{3}$, find $\cos \theta$.
$\frac{3}{5}$ | 6. If $\sin \theta = \frac{3}{7}$, find $\tan \theta$.
$\frac{3\sqrt{10}}{20}$ |

Find the exact value of each expression if $90^\circ < \theta < 180^\circ$.

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|---|--|
| 7. If $\cos \theta = -\frac{7}{8}$, find $\sec \theta$.
$-\frac{8}{7}$ | 8. If $\csc \theta = \frac{12}{5}$, find $\cot \theta$.
$-\frac{\sqrt{119}}{5}$ |
|---|--|

Find the exact value of each expression if $270^\circ < \theta < 360^\circ$.

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|--|--|
| 9. If $\cos \theta = \frac{6}{7}$, find $\sin \theta$.
$-\frac{\sqrt{13}}{7}$ | 10. If $\csc \theta = -\frac{9}{4}$, find $\sin \theta$.
$-\frac{4}{9}$ |
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14-1 Study Guide and Intervention *(continued)***Trigonometric Identities**

Simplify Expressions The simplified form of a trigonometric expression is written as a numerical value or in terms of a single trigonometric function, if possible. Any of the trigonometric identities can be used to simplify expressions containing trigonometric functions.

Example 1 Simplify $(1 - \cos^2 \theta) \sec \theta \cot \theta + \tan \theta \sec \theta \cos^2 \theta$.

$$\begin{aligned} (1 - \cos^2 \theta) \sec \theta \cot \theta + \tan \theta \sec \theta \cos^2 \theta &= \sin^2 \theta \cdot \frac{1}{\cos \theta} \cdot \frac{\cos \theta}{\sin \theta} + \frac{\sin \theta}{\cos \theta} \cdot \frac{1}{\cos \theta} \cdot \cos^2 \theta \\ &= \sin \theta + \sin \theta \\ &= 2 \sin \theta \end{aligned}$$

Example 2 Simplify $\frac{\sec \theta \cdot \cot \theta}{1 - \sin \theta} - \frac{\csc \theta}{1 + \sin \theta}$.

$$\begin{aligned} \frac{\sec \theta \cdot \cot \theta}{1 - \sin \theta} - \frac{\csc \theta}{1 + \sin \theta} &= \frac{\frac{1}{\cos \theta} \cdot \frac{\cos \theta}{\sin \theta}}{1 - \sin \theta} - \frac{\frac{1}{\sin \theta}}{1 + \sin \theta} \\ &= \frac{\frac{1}{\sin \theta}(1 + \sin \theta) - \frac{1}{\sin \theta}(1 - \sin \theta)}{(1 - \sin \theta)(1 + \sin \theta)} \\ &= \frac{\frac{1}{\sin \theta} + 1 - \frac{1}{\sin \theta} + 1}{1 - \sin^2 \theta} \\ &= \frac{2}{\cos^2 \theta} \end{aligned}$$

Exercises

Simplify each expression.

1. $\frac{\tan \theta \cdot \csc \theta}{\sec \theta} \quad \mathbf{1}$

2. $\frac{\sin \theta \cdot \cot \theta}{\sec^2 \theta - \tan^2 \theta} \quad \mathbf{\cos \theta}$

3. $\frac{\sin^2 \theta - \cot \theta \cdot \tan \theta}{\cot \theta \cdot \sin \theta} \quad \mathbf{-\cos \theta}$

4. $\frac{\cos \theta}{\sec \theta - \tan \theta} \quad \mathbf{1 + \sin \theta}$

5. $\frac{\tan \theta \cdot \cos \theta}{\sin \theta} + \cot \theta \cdot \sin \theta \cdot \tan \theta \cdot \csc \theta \quad \mathbf{2}$

6. $\frac{\csc^2 \theta - \cot^2 \theta}{\tan \theta \cdot \cos \theta} \quad \mathbf{\csc \theta}$

7. $3 \tan \theta \cdot \cot \theta + 4 \sin \theta \cdot \csc \theta + 2 \cos \theta \cdot \sec \theta \quad \mathbf{9}$

8. $\frac{1 - \cos^2 \theta}{\tan \theta \cdot \sin \theta} \quad \mathbf{\cos \theta}$